
Three-Level Calculations

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This Mathcad sheet plots waveforms for a three-level inverter. Individual transistor and diode waveforms are created for calculation of switching and conduction losses as well as peak and average currents.

Inverter ratings

$$\begin{aligned}v_{dc} &:= 650 \cdot V & i_s &:= 100A & pf &:= 0.8 & f &:= 60 \cdot \text{Hz} \\ m &:= 0.95 & f_{sw} &:= 5 \cdot \text{kHz}\end{aligned}$$

transistor ratings

$$\begin{aligned}v_{IGBT} &:= \frac{v_{dc}}{2} & v_{IGBT} &= 325 \text{ V} \\ i_{IGBT} &:= \sqrt{2} \cdot i_s & i_{IGBT} &= 141 \text{ A}\end{aligned}$$

phase current

$$\begin{aligned}\phi_i &:= -\arccos(pf) & \left(\frac{180}{\pi}\right) \cdot \phi_i &= -36.87 \\ i_{as}(\theta) &:= \sqrt{2} \cdot i_s \cdot \cos(\theta + \phi_i) \\ I_a(\theta) &:= \text{if}(i_{as}(\theta) > 0, 1, 0)\end{aligned}$$

power factor check

$$\tau := \frac{-\tan(\phi_i)}{2 \cdot \pi \cdot f} \quad \tau = 1.989 \times 10^{-3} \text{ s}$$

$$T_{sw} := \frac{1}{f_{sw}} \quad T_{sw} = 200 \times 10^{-6} \text{ s}$$

$$\frac{\tau}{T_{sw}} = 9.947 \quad \text{<--- ratio of load time constant to switching period}$$

This should be significant for accuracy

duty cycle

$$d_{\text{am}}(\theta) := 1 + m \cdot \cos(\theta)$$

triangle waveforms

$$tr_1(\theta) := \frac{1}{2} + \frac{4}{\pi^2} \cdot \left[\sum_{n=1}^5 \frac{\cos \left[(2 \cdot n - 1) \cdot \theta \cdot \frac{f_{\text{sw}}}{f} \right]}{(2 \cdot n - 1)^2} \right]$$

$$tr_2(\theta) := \frac{3}{2} + \frac{-4}{\pi^2} \cdot \left[\sum_{n=1}^5 \frac{\cos \left[(2 \cdot n - 1) \cdot \theta \cdot \frac{f_{\text{sw}}}{f} \right]}{(2 \cdot n - 1)^2} \right]$$

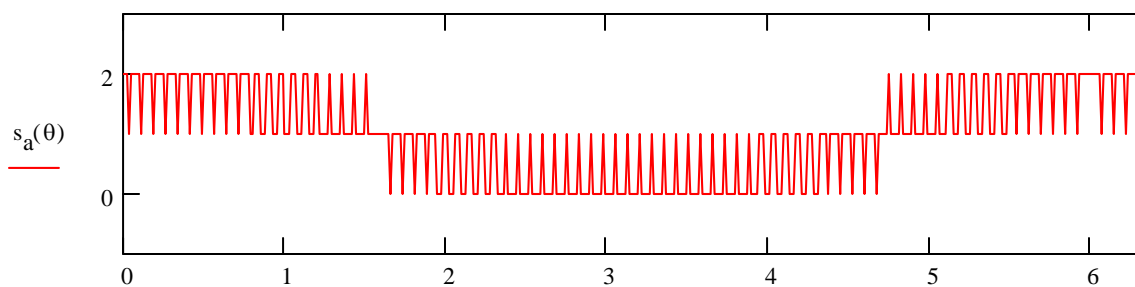
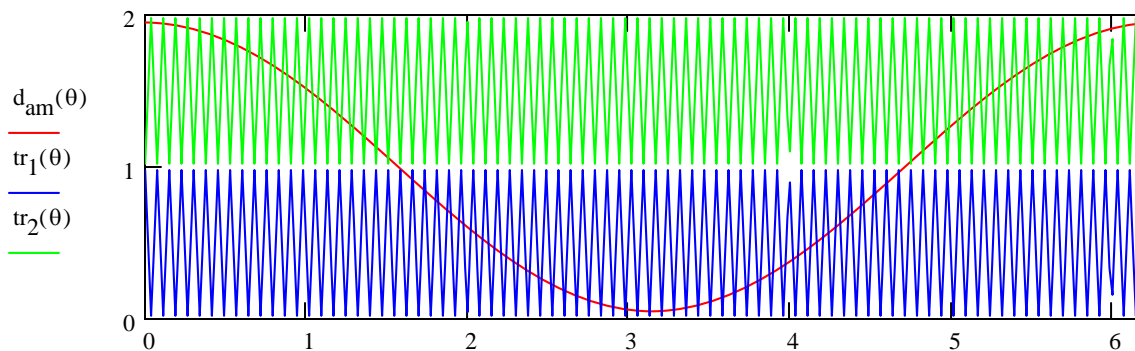
switching functions

$$s_a(\theta) := \text{if}(d_{\text{am}}(\theta) > tr_1(\theta), 1, 0) + \text{if}(d_{\text{am}}(\theta) > tr_2(\theta), 1, 0)$$

$$s_{a0}(\theta) := \text{if}(s_a(\theta) = 0, 1, 0)$$

$$s_{a1}(\theta) := \text{if}(s_a(\theta) = 1, 1, 0)$$

$$s_{a2}(\theta) := \text{if}(s_a(\theta) = 2, 1, 0)$$



line-to-ground voltage

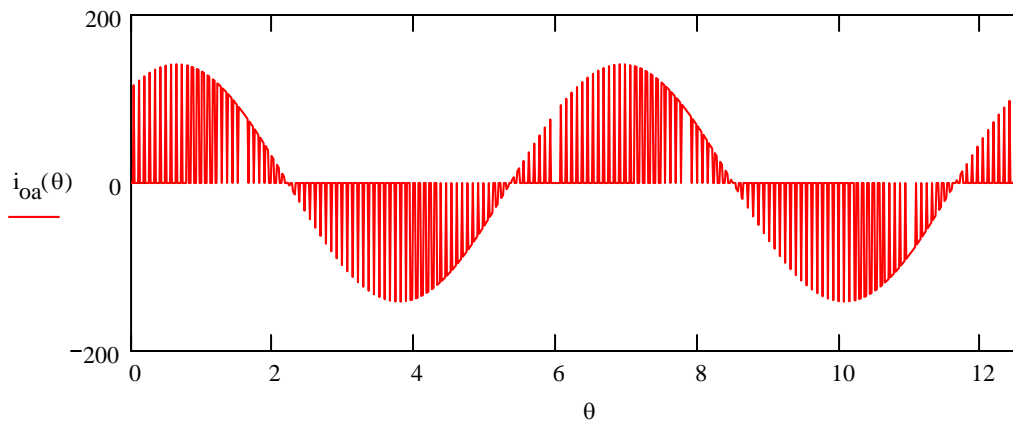
$$v_{ag}(\theta) := \frac{v_{dc}}{2} \cdot s_a(\theta)$$

neutral-point current (a-phase contribution)

$$i_{oa}(\theta) := s_{a1}(\theta) \cdot i_{as}(\theta)$$

$$i_{c2a}(\theta) := s_{a2}(\theta) \cdot i_{as}(\theta)$$

$$i_{c1a}(\theta) := s_{a0}(\theta) \cdot i_{as}(\theta)$$



$$t_{\min} := 20 \cdot 10^{-6} \cdot \text{sec}$$

$$n := \frac{2}{6 \cdot f \cdot t_{\min}} \quad n = 278$$

$$i := 0, 1 \dots (n - 1)$$

$$i_{ca_rms} := \sqrt{\frac{3}{\pi \cdot 2 \cdot n} \sum_i \left(i_{c1a} \left(\frac{i}{n-1} \cdot \frac{\pi \cdot 2}{3} \right) \right)^2} \quad i_{ca_rms} = 7.1 \text{ A}$$

transistor and diode voltages

$$v_{Ta1}(\theta) := \frac{v_{dc}}{2} \cdot (1 - s_{a2}(\theta))$$

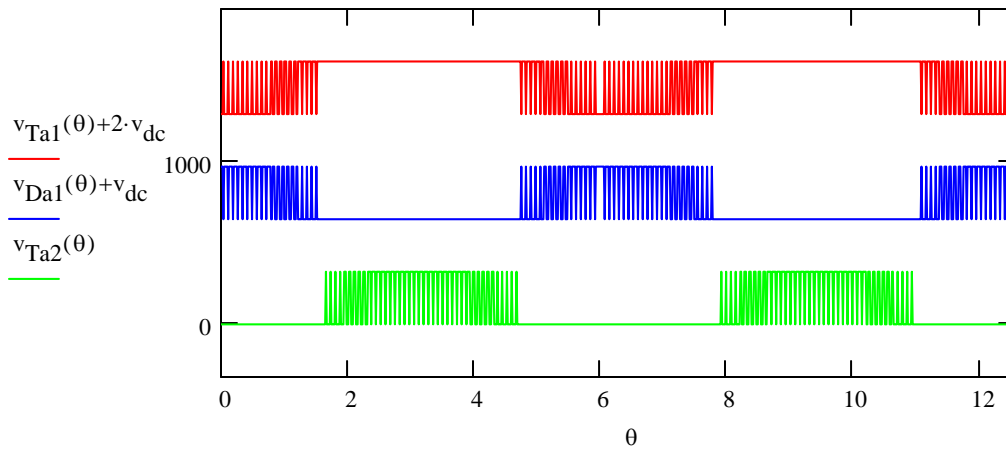
$$v_{Da1}(\theta) := \frac{v_{dc}}{2} \cdot s_{a2}(\theta)$$

$$v_{Ta2}(\theta) := \frac{v_{dc}}{2} \cdot s_{a0}(\theta)$$

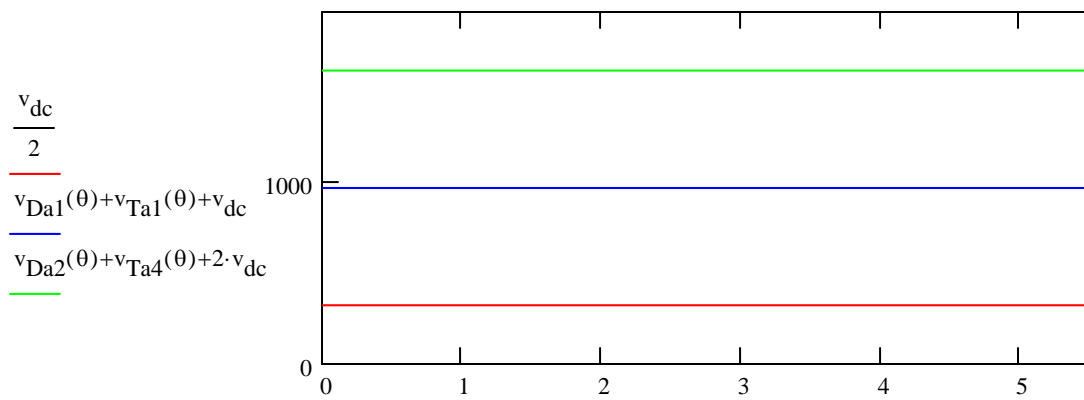
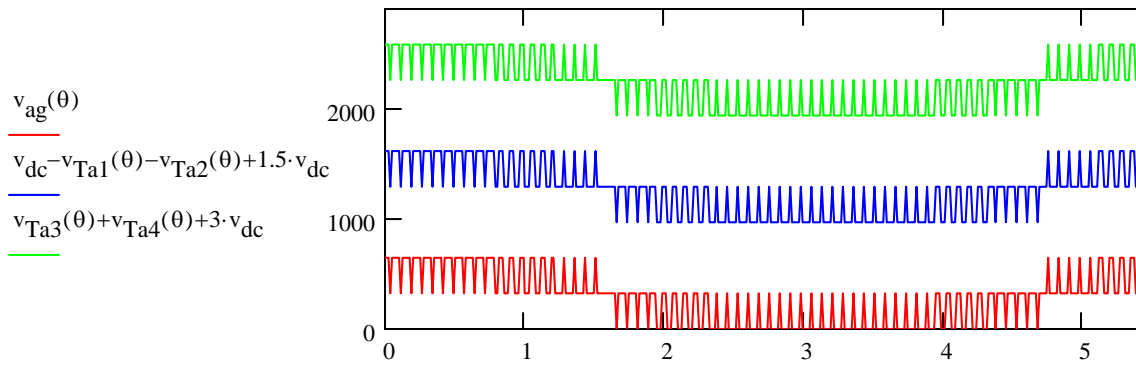
$$v_{Ta3}(\theta) := \frac{v_{dc}}{2} \cdot s_{a2}(\theta)$$

$$v_{Da2}(\theta) := \frac{v_{dc}}{2} \cdot s_{a0}(\theta)$$

$$v_{Ta4}(\theta) := \frac{v_{dc}}{2} \cdot (1 - s_{a0}(\theta))$$



diode and transistor voltage checksum



$$\frac{v_{dc}}{2} = 325 \text{ V}$$

$$v_{Da1}(0) + v_{Ta1}(0) = 325 \text{ V}$$

$$v_{Da2}(0) + v_{Ta4}(0) = 325 \text{ V}$$

transistor and diode currents

$$i_{Ta1}(\theta) := i_{as}(\theta) \cdot I_a(\theta) \cdot s_{a2}(\theta)$$

$$i_{Ta1FD}(\theta) := -i_{as}(\theta) \cdot (1 - I_a(\theta)) \cdot s_{a2}(\theta)$$

$$i_{Da1}(\theta) := i_{as}(\theta) \cdot I_a(\theta) \cdot s_{a1}(\theta)$$

$$i_{Ta2}(\theta) := i_{as}(\theta) \cdot I_a(\theta) \cdot (1 - s_{a0}(\theta))$$

$$i_{Ta2FD}(\theta) := -i_{as}(\theta) \cdot (1 - I_a(\theta)) \cdot s_{a2}(\theta)$$

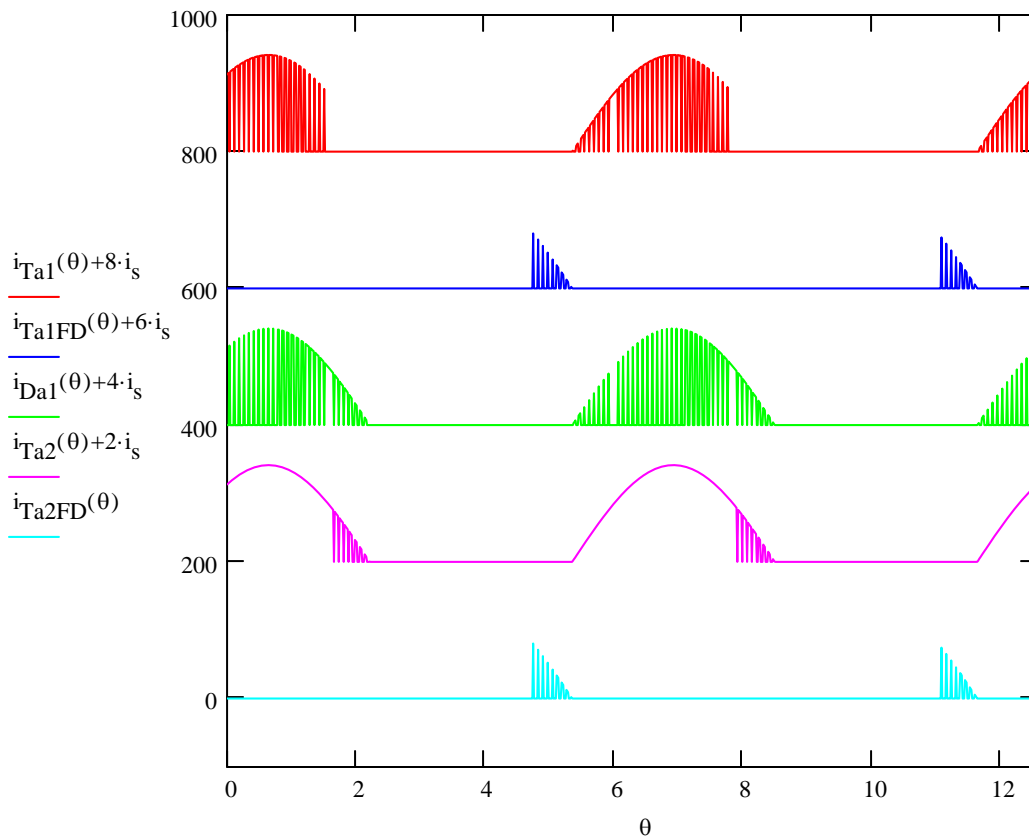
$$i_{Ta3}(\theta) := -i_{as}(\theta) \cdot (1 - I_a(\theta)) \cdot (1 - s_{a2}(\theta))$$

$$i_{Ta3FD}(\theta) := i_{as}(\theta) \cdot I_a(\theta) \cdot s_{a0}(\theta)$$

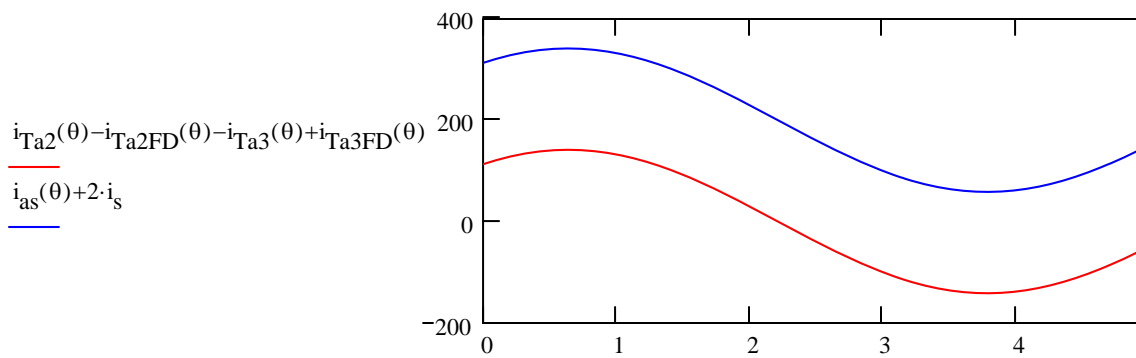
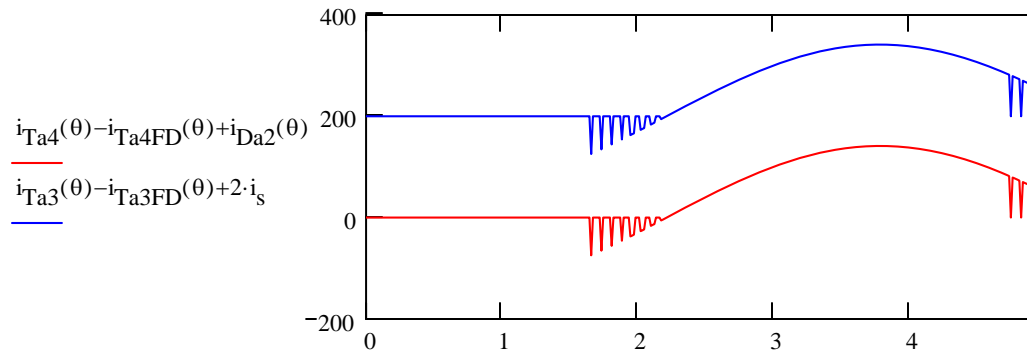
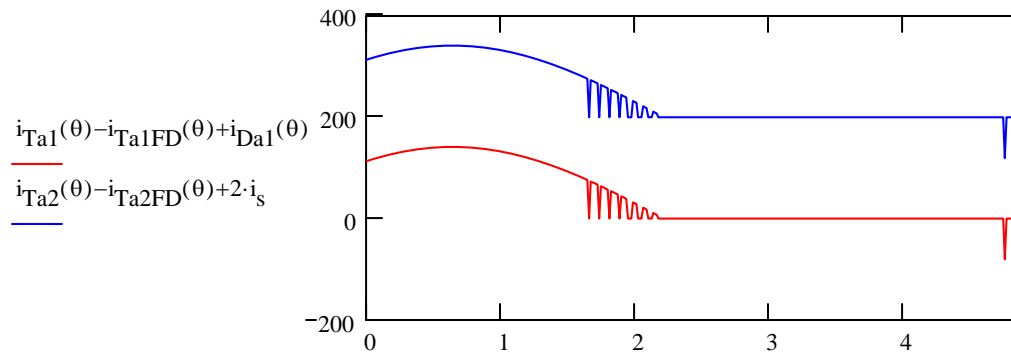
$$i_{Da2}(\theta) := -i_{as}(\theta) \cdot (1 - I_a(\theta)) \cdot s_{a1}(\theta)$$

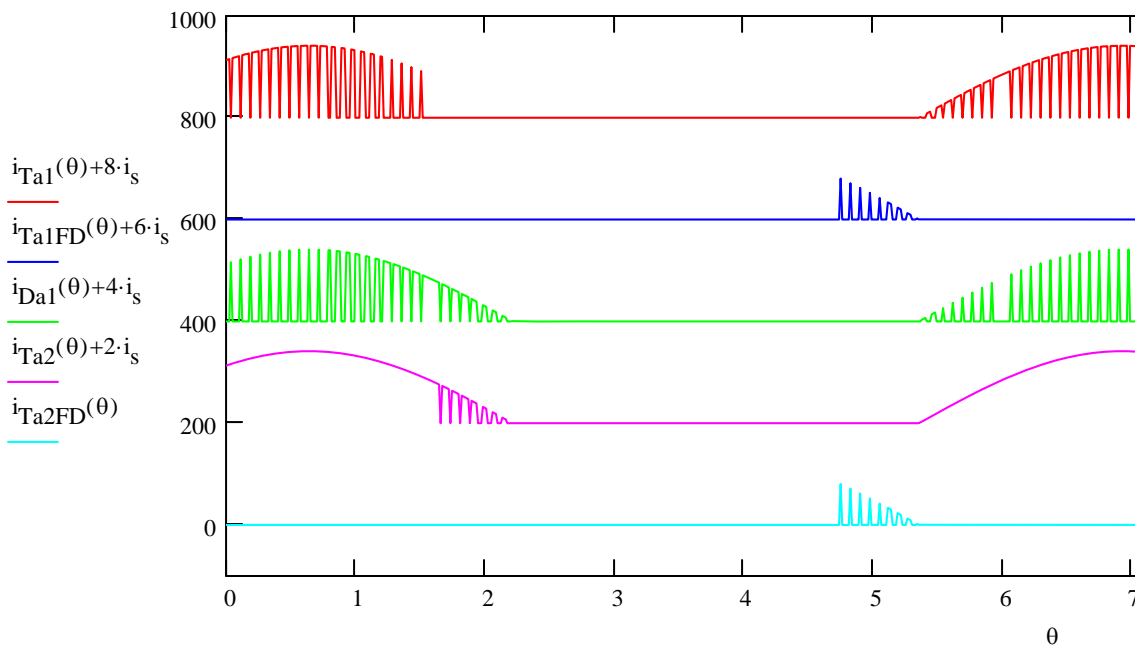
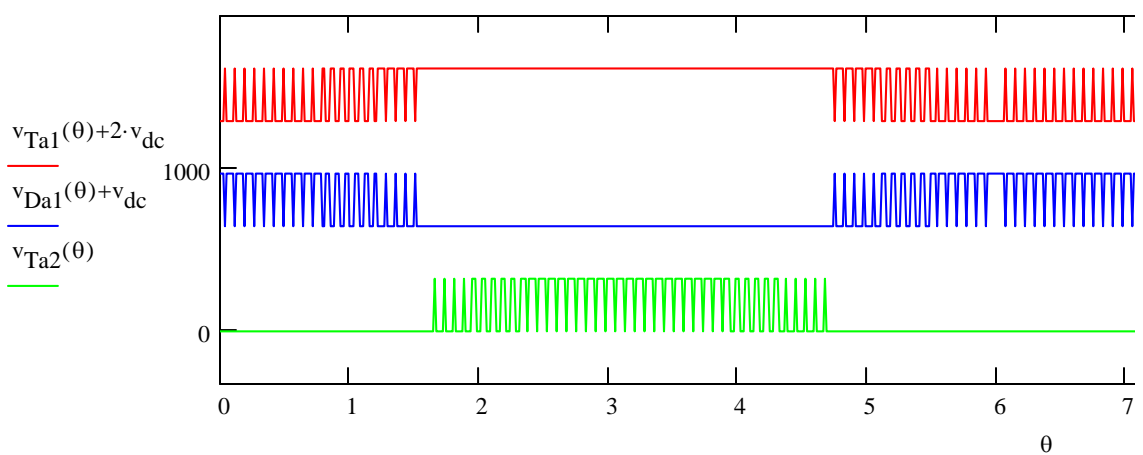
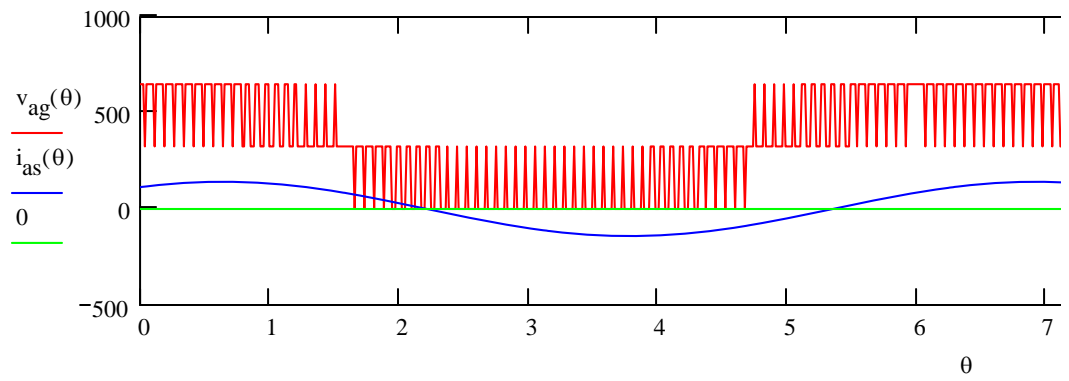
$$i_{Ta4}(\theta) := -i_{as}(\theta) \cdot (1 - I_a(\theta)) \cdot s_{a0}(\theta)$$

$$i_{Ta4FD}(\theta) := i_{as}(\theta) \cdot I_a(\theta) \cdot s_{a0}(\theta)$$



transistor and diode current checksum





average currents and conduction losses

$$V_Q := 2 \cdot \text{volt} \quad V_D := 1.5 \cdot \text{volt}$$

$$t_{\min} := 20 \cdot 10^{-6} \cdot \text{sec}$$

$$n := \frac{1}{f \cdot t_{\min}} \quad n = 833$$

$$i := 0, 1 \dots (n - 1)$$

$$i_{\text{Ta1_avg}} := \left(\frac{1}{n - 1} \right) \cdot \sum_i i_{\text{Ta1}} \left(\frac{i \cdot 2 \cdot \pi}{n - 1} \right) \quad i_{\text{Ta1_avg}} = 27.62 \text{ A}$$

$$i_{\text{Ta1FD_avg}} := \left(\frac{1}{n - 1} \right) \cdot \sum_i i_{\text{Ta1FD}} \left(\frac{i \cdot 2 \cdot \pi}{n - 1} \right) \quad i_{\text{Ta1FD_avg}} = 0.93 \text{ A}$$

$$i_{\text{Da1_avg}} := \left(\frac{1}{n - 1} \right) \cdot \sum_i i_{\text{Da1}} \left(\frac{i \cdot 2 \cdot \pi}{n - 1} \right) \quad i_{\text{Da1_avg}} = 16.54 \text{ A}$$

$$i_{\text{Ta2_avg}} := \left(\frac{1}{n - 1} \right) \cdot \sum_i i_{\text{Ta2}} \left(\frac{i \cdot 2 \cdot \pi}{n - 1} \right) \quad i_{\text{Ta2_avg}} = 44.15 \text{ A}$$

$$i_{\text{Ta2FD_avg}} := \left(\frac{1}{n - 1} \right) \cdot \sum_i i_{\text{Ta2FD}} \left(\frac{i \cdot 2 \cdot \pi}{n - 1} \right) \quad i_{\text{Ta2FD_avg}} = 0.93 \text{ A}$$

$$P_{\text{cond_Ta1}} := i_{\text{Ta1_avg}} \cdot V_Q \quad P_{\text{cond_Ta1}} = 55.23 \text{ W}$$

$$P_{\text{cond_Ta1FD}} := i_{\text{Ta1FD_avg}} \cdot V_D \quad P_{\text{cond_Ta1FD}} = 1.4 \text{ W}$$

$$P_{\text{cond_Da1}} := i_{\text{Da1_avg}} \cdot V_D \quad P_{\text{cond_Da1}} = 24.81 \text{ W}$$

$$P_{\text{cond_Ta2}} := i_{\text{Ta2_avg}} \cdot V_Q \quad P_{\text{cond_Ta2}} = 88.31 \text{ W}$$

$$P_{\text{cond_Ta2FD}} := i_{\text{Ta2FD_avg}} \cdot V_D \quad P_{\text{cond_Ta2FD}} = 1.4 \text{ W}$$

$$P_{\text{cond}} := 6 \cdot P_{\text{cond_Ta1}} + 6 \cdot P_{\text{cond_Ta1FD}} + 6 \cdot P_{\text{cond_Da1}} \dots \\ + 6 \cdot P_{\text{cond_Ta2}} + 6 \cdot P_{\text{cond_Ta2FD}}$$

$P_{\text{cond}} = 1027 \text{ W}$

switching losses

$$E_{\text{on}} := 0.13 \cdot \text{watt} \cdot \text{sec} \quad E_{\text{off}} := 0.13 \cdot \text{watt} \cdot \text{sec} \quad v_E := 1200 \cdot \text{volt} \quad i_E := 300 \cdot \text{amp}$$

$$E_{\text{fr}} := 0 \cdot \text{watt} \cdot \text{sec} \quad E_{\text{rr}} := 0.13 \cdot \text{watt} \cdot \text{sec}$$

$$E_t := 0.5 \cdot (E_{\text{on}} + E_{\text{off}}) \quad E_t = 0.13 \text{ J}$$

$$E_d := 0.5 \cdot (E_{\text{fr}} + E_{\text{rr}}) \quad E_d = 0.065 \text{ J}$$

$$t_{\text{min}} := 5 \cdot 10^{-6} \cdot \text{sec}$$

$$n := \frac{1}{f \cdot t_{\text{min}}} \quad n = 3333$$

$$i := 0, 1 \dots (n - 1)$$

$$\Delta\theta := \frac{2 \cdot \pi}{n - 1}$$

$$s_{\text{Ta1}} := \text{if} \left[\begin{array}{l} i_{\text{Ta1}}(i \cdot \Delta\theta) = 0 \wedge i_{\text{Ta1}}[(i + 1) \cdot \Delta\theta] \neq 0, 1, 0 \dots \\ + \text{if} \left[i_{\text{Ta1}}(i \cdot \Delta\theta) \neq 0 \wedge i_{\text{Ta1}}[(i + 1) \cdot \Delta\theta] = 0, 1, 0 \right] \end{array} \right] \dots$$

$$j := \text{match}(1, s_{\text{Ta1}})$$

$$m := \text{rows}(j)$$

$$k := 0, 1 \dots (m - 1)$$

$$P_{\text{sw_Ta1}} := \frac{f \cdot E_t}{v_E \cdot i_E} \cdot \sum_k \left[\begin{array}{l} v_{\text{Ta1}}(j_k \cdot \Delta\theta) \cdot i_{\text{Ta1}}[(j_k + 1) \cdot \Delta\theta] \dots \\ + v_{\text{Ta1}}[(j_k + 1) \cdot \Delta\theta] \cdot i_{\text{Ta1}}(j_k \cdot \Delta\theta) \end{array} \right] \quad P_{\text{sw_Ta1}} = 46.6 \text{ W}$$

$$s_{\text{Ta1FD}} := \text{if} \left[\begin{array}{l} i_{\text{Ta1FD}}(i \cdot \Delta\theta) = 0 \wedge i_{\text{Ta1FD}}[(i + 1) \cdot \Delta\theta] \neq 0, 1, 0 \dots \\ + \text{if} \left[i_{\text{Ta1FD}}(i \cdot \Delta\theta) \neq 0 \wedge i_{\text{Ta1FD}}[(i + 1) \cdot \Delta\theta] = 0, 1, 0 \right] \end{array} \right] \dots$$

$$j := \text{match}(1, s_{\text{Ta1FD}})$$

$$m := \text{rows}(j)$$

$$k := 0, 1 \dots (m - 1)$$

$$P_{\text{sw_Ta1FD}} := \frac{f \cdot E_d}{v_E \cdot i_E} \cdot \sum_k \left[\begin{array}{l} v_{\text{Ta1}}(j_k \cdot \Delta\theta) \cdot i_{\text{Ta1FD}}[(j_k + 1) \cdot \Delta\theta] \dots \\ + v_{\text{Ta1}}[(j_k + 1) \cdot \Delta\theta] \cdot i_{\text{Ta1FD}}(j_k \cdot \Delta\theta) \end{array} \right] \quad P_{\text{sw_Ta1FD}} = 2.6 \text{ W}$$

switching losses (continued)

$$s_{Da1_i} := \text{if} \left[i_{Da1}(i \cdot \Delta\theta) = 0 \wedge i_{Da1}[(i+1) \cdot \Delta\theta] \neq 0, 1, 0 \right] \dots \\ + \text{if} \left[i_{Da1}(i \cdot \Delta\theta) \neq 0 \wedge i_{Da1}[(i+1) \cdot \Delta\theta] = 0, 1, 0 \right]$$

$$j := \text{match}(1, s_{Da1})$$

$$m := \text{rows}(j)$$

$$k := 0, 1 \dots (m-1)$$

$$P_{sw_Da1} := \frac{f \cdot E_d}{v_E \cdot i_E} \sum_k \left[v_{Da1}(j_k \cdot \Delta\theta) \cdot i_{Da1}[(j_k+1) \cdot \Delta\theta] \dots \right. \\ \left. + v_{Da1}[(j_k+1) \cdot \Delta\theta] \cdot i_{Da1}(j_k \cdot \Delta\theta) \right]$$

$$P_{sw_Da1} = 23.3 \text{ W}$$

$$s_{Ta2_i} := \text{if} \left[i_{Ta2}(i \cdot \Delta\theta) = 0 \wedge i_{Ta2}[(i+1) \cdot \Delta\theta] \neq 0, 1, 0 \right] \dots \\ + \text{if} \left[i_{Ta2}(i \cdot \Delta\theta) \neq 0 \wedge i_{Ta2}[(i+1) \cdot \Delta\theta] = 0, 1, 0 \right]$$

$$j := \text{match}(1, s_{Ta2})$$

$$m := \text{rows}(j)$$

$$k := 0, 1 \dots (m-1)$$

$$P_{sw_Ta2} := \frac{f \cdot E_t}{v_E \cdot i_E} \sum_k \left[v_{Ta2}(j_k \cdot \Delta\theta) \cdot i_{Ta2}[(j_k+1) \cdot \Delta\theta] \dots \right. \\ \left. + v_{Ta2}[(j_k+1) \cdot \Delta\theta] \cdot i_{Ta2}(j_k \cdot \Delta\theta) \right]$$

$$P_{sw_Ta2} = 4.5 \text{ W}$$

$$s_{Ta2FD_i} := \text{if} \left[i_{Ta2FD}(i \cdot \Delta\theta) = 0 \wedge i_{Ta2FD}[(i+1) \cdot \Delta\theta] \neq 0, 1, 0 \right] \dots \\ + \text{if} \left[i_{Ta2FD}(i \cdot \Delta\theta) \neq 0 \wedge i_{Ta2FD}[(i+1) \cdot \Delta\theta] = 0, 1, 0 \right]$$

$$j := \text{match}(1, s_{Ta2FD})$$

$$m := \text{rows}(j)$$

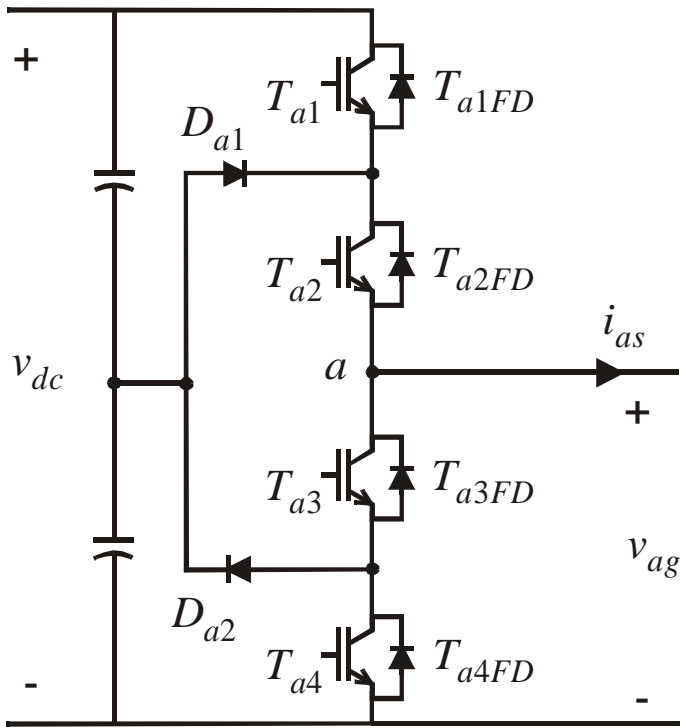
$$k := 0, 1 \dots (m-1)$$

$$P_{sw_Ta2FD} := \frac{f \cdot E_d}{v_E \cdot i_E} \sum_k \left[v_{Ta2}(j_k \cdot \Delta\theta) \cdot i_{Ta2FD}[(j_k+1) \cdot \Delta\theta] \dots \right. \\ \left. + v_{Ta2}[(j_k+1) \cdot \Delta\theta] \cdot i_{Ta2FD}(j_k \cdot \Delta\theta) \right]$$

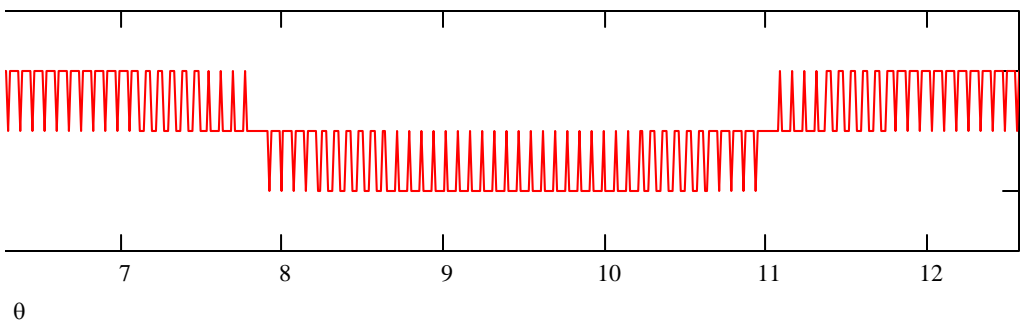
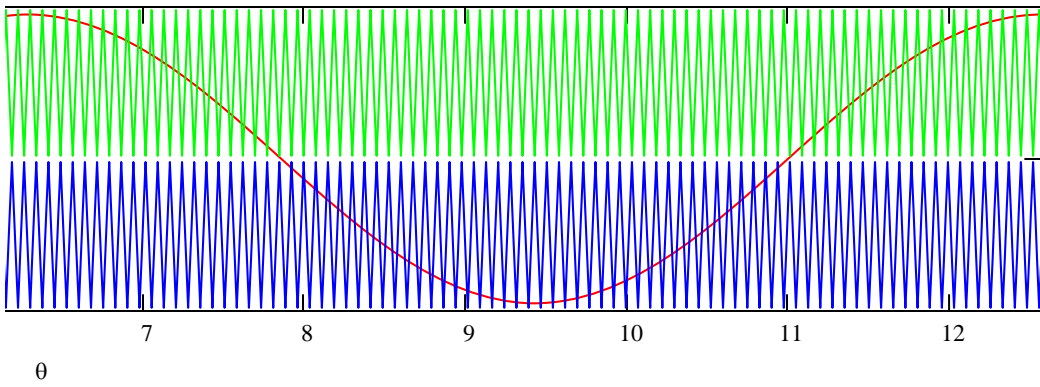
$$P_{sw_Ta2FD} = 0 \text{ W}$$

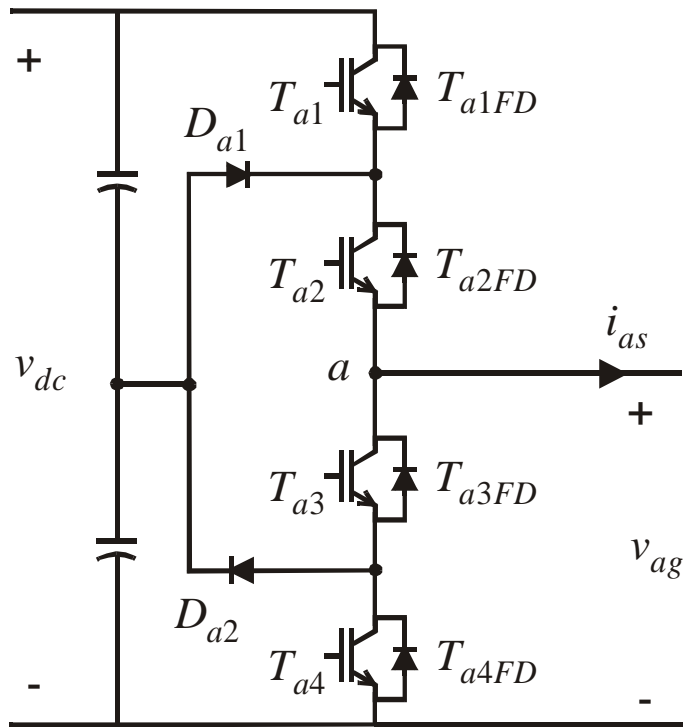
$$P_{sw} := 6 \cdot P_{sw_Ta1} + 6 \cdot P_{sw_Ta1FD} + 6 \cdot P_{sw_Da1} \dots \\ + 6 \cdot P_{sw_Ta2} + 6 \cdot P_{sw_Ta2FD}$$

$$P_{sw} = 462 \text{ W}$$

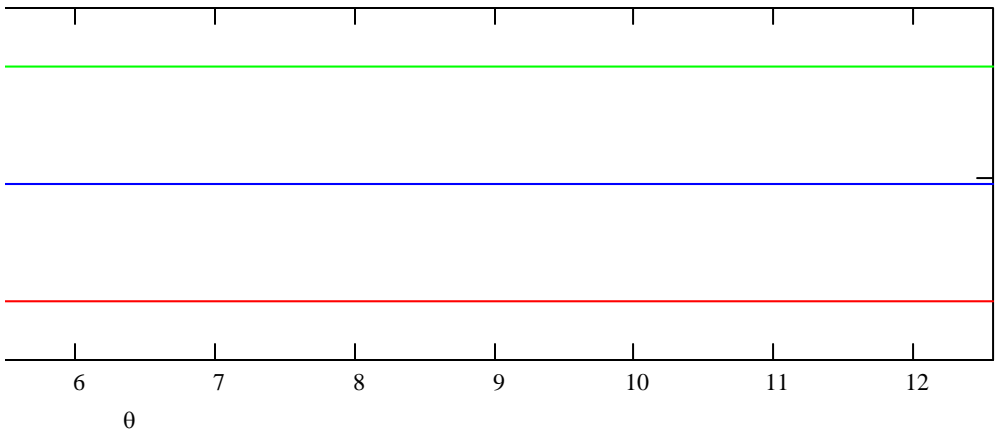
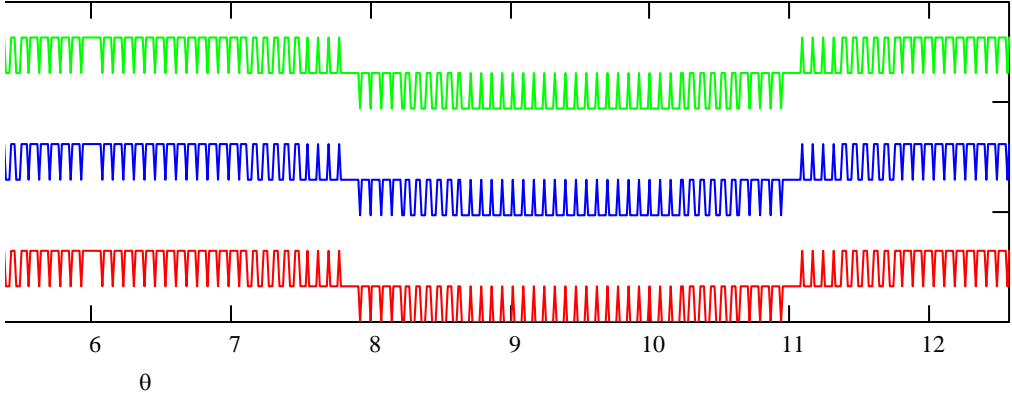


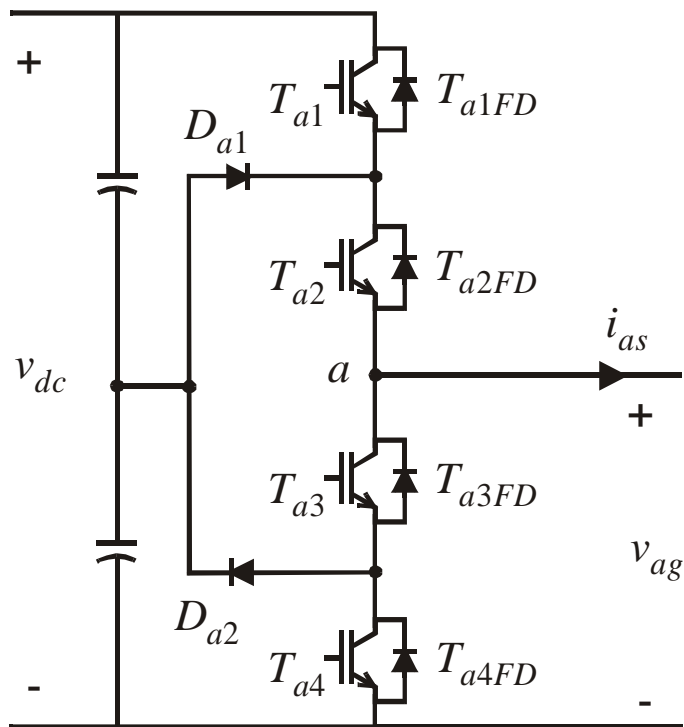
Three-level diode-clamped topology
(a-phase)



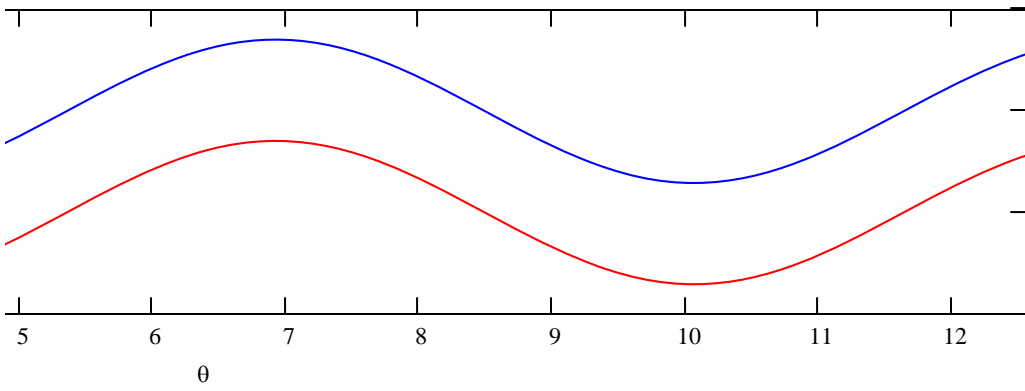
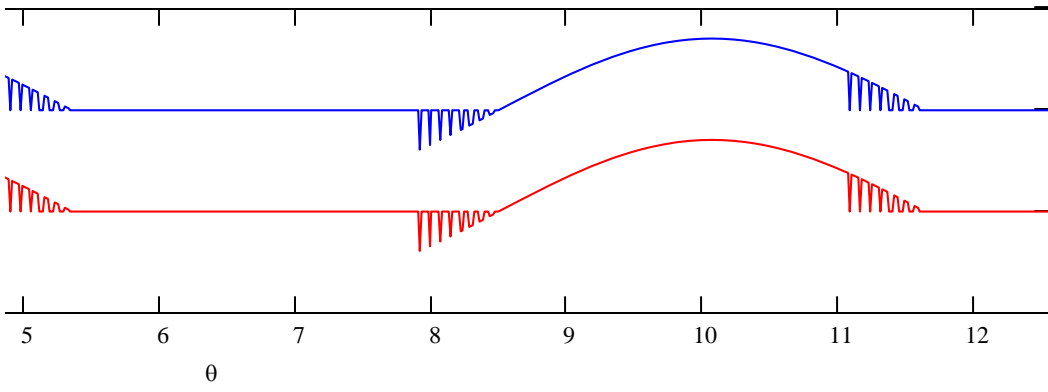
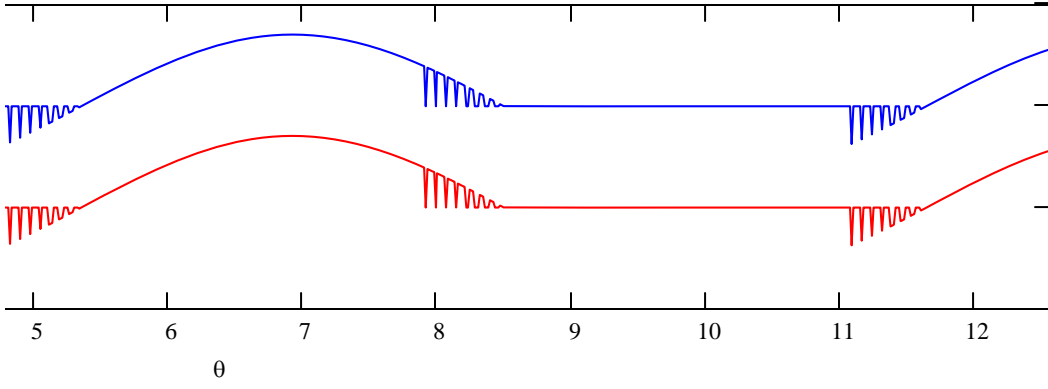


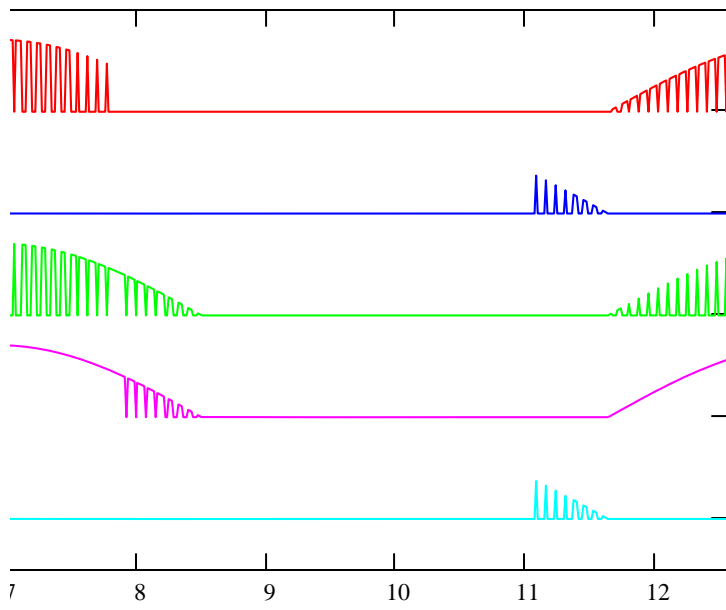
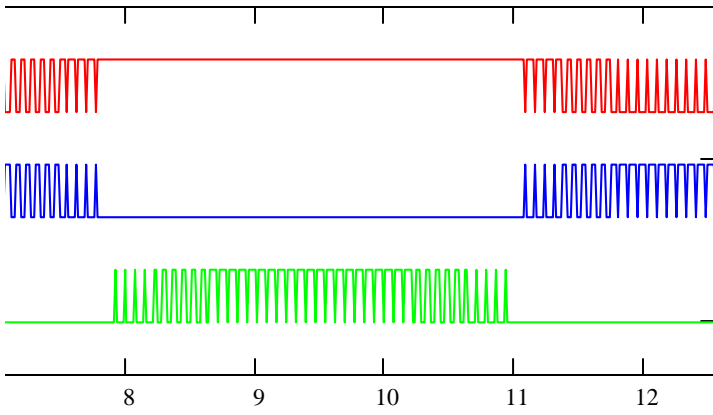
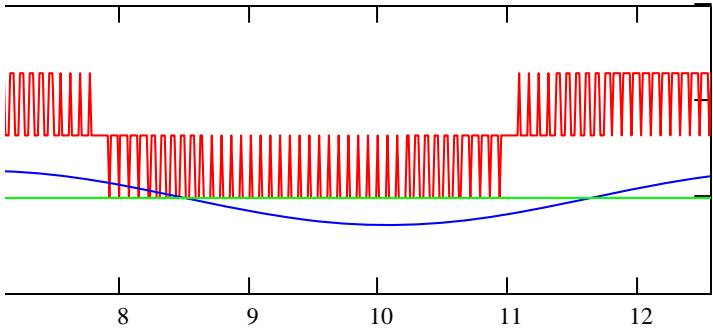
Three-level diode-clamped topology
(a-phase)





Three-level diode-clamped topology
(a-phase)





$$i_{\text{Tal}_i} := i_{\text{Tal}} \left(\frac{i \cdot 2 \cdot \pi}{n - 1} \right)$$

$$i_{\text{Tal}_{\text{max}}} := \max(i_{\text{Tal}_i})$$

$$i_{\text{Tal}_{\text{max}}} = 141 \text{ A}$$

$$i_{\text{Tal}_{\text{avg}}} := \left(\frac{1}{n - 1} \right) \cdot \sum_i i_{\text{Tal}_i}$$

$$i_{\text{Tal}_{\text{avg}}} = 27.62 \text{ A}$$